A Binary Sequence of Period 60 With Better Autocorrelation Properties Than the Barker Sequence of Period 13

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A binary sequence of period 60 has been discovered which in some respects has better autocorrelation properties than the Barker sequence of period 13. When both sequences are processed using appropriate sidelobe-eliminating mismatched filters, the Barker sequence's main lobe is reduced by a factor of 1.040 or 0.17 dB, while the new sequence's main lobe is reduced by a factor of only 1.035 or 0.15 dB. This sequence is the first counterexample known to the authors of the hypothesis that the autocorrelation properties of all sequences of periods greater than 13 are inferior to those of the Barker period-13 sequence. Sequences of this type are very useful in radar and deep space communications, especially in situations where there is an adverse signal to noise ratio.

I. Introduction

A binary sequence is a string of bits. It can be thought of as a vector $\boldsymbol{C}$ where each C_i is a plus one or a minus one. A periodic sequence is one which is continuously repeated; for a binary sequence of period j:

$$C_{i+j} = C_i$$

for all i. The autocorrelation, $\boldsymbol{a}$, of a sequence, $\boldsymbol{C}$, of period j is:

$$a_i = \sum_{k=1}^{j} C_K C_{K+i}$$

The element a_j is called the main lobe; the remaining a_i are sidelobes. For example, if $\boldsymbol{C} = (1, 1, 1, -1)$, $\boldsymbol{a} = (0, 0, 0, 4)$. In this example, the sidelobes are zero. For a sequence to have good autocorrelation properties the sidelobes should be small. The example used is perfect, but for sequence periods other than 4, the autocorrelation sidelobes are never all zeros.

The cross-correlation, $\boldsymbol{X}$, of two sequences, $\boldsymbol{B}$ and $\boldsymbol{C}$, each of period j is

$$X_i = \sum_{K=1}^{j} B_K C_{K+i}$$

Let $\boldsymbol{C}$ be a binary sequence which is to be cross-correlated with $\boldsymbol{B}$, a sequence composed of real numbers. The sequences $\boldsymbol{C}$ and $\boldsymbol{B}$ are related by the weighting function, $\boldsymbol{T}$.

$$B_i = T_i C_i$$

In this case, $\boldsymbol{B}$ can be considered a "mismatched filter" to $\boldsymbol{C}$.

For a mismatched filter $\boldsymbol{B}$ to be normalized,

$$\sum_{i=1}^{j} B_i^2 = j$$

Starting with a binary sequence, for example;

$$C_1 = (1, -1, 1, -1, -1, -1, -1, -1, 1, 1, 1, -1, -1, 1, -1, -1)$$

and the (normalized) weighting function,

$$T_i = \sqrt{2/11}\,(1, 4, 1, 2, 1, 4, 1, 2, 1, 4, 1, 2, 1, 4, 1, 2)$$

then the normalized mismatched filter

$$B_i = \sqrt{2/11}$$

$$\times\,(1, -4, 1, -2, -1, -4, -1, -2, 1, 4, 1, -2, -1, 4, -1, -2)$$

and the cross-correlation

$$X_i = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \sqrt{2048/11})$$

Examples of periodic binary sequences with good autocorrelation properties are available from a number of sources as shown in the companion paper to this article (Ref. 1). Such sequences are utilized extensively in satellite and planetary probe ranging and communications (Ref. 2). Mismatched weighting functions can often suppress the cross-correlation sidelobes to arbitrarily low levels, but only at the expense of a loss in signal to noise ratio (SNR) (Ref. 3).

This loss equals the square of the sequence period divided by the square of the main lobe. In the above example of a sequence of period 16 analyzed with a normalized mismatched filter to give no sidelobes, the loss is:

$$\text{loss} = 256 \times \frac{11}{2048} = \frac{11}{8} = 1.375$$

The loss in dB is $10 \log_{10} (1.375) = 1.38$ dB. A method for calculating sidelobe-eliminating weighting functions is given in Ref. 1.

A periodic sequence is a Barker sequence if its sidelobes are all +1. A Barker sequence can have two periods, 5:

+ + + + -

and 13:

+ + + + + - - + + - + - +

The Barker sequence of period 13 has a reduction in main lobe of a factor of 1.040 when processed by a mismatched filter which completely suppresses all cross-correlation sidelobes, corresponding to a loss in signal to noise ratio of 0.17 dB (Ref. 4). Until now, the authors had not been able to discover a counterexample to the hypothesis that no sequence of period greater than 13 will have better autocorrelation properties than those of this Barker sequence. For example, the best codes listed for periods 36, 39, and 64 all have somewhat greater SNR losses (0.19, 0.22, and 0.23 dB, respectively) than does the Barker code of period 13.

II. A New Sequence of Period 60

A search of binary sequences of period 60 was performed on a Cyber 750 computer. The Cyber was chosen due to its availability, speed, and the inclusion in its assembly language of an instruction which counts the number of ones in a word. This latter property greatly facilitates a search for sequences which have good matched-filter autocorrelation properties (Ref. 5). The period was chosen to correspond to the Cyber's word length of 60. Good sequences were found iteratively by looking at all one-bit modifications of the previous best sequence.

The sequence evaluator, written in COMPASS, was decisive in permitting an adequate number of sequences to be evaluated in a reasonable amount of time. The program segment in Fig. 1 was used for this task.

The Cyber processed approximately 10^8 sequences per hour. Within 50 computer-hours, two sequences were discovered each having an excellent matched-filter autocorrelation. One of them is:

+ + - + - + + - - + - + - - - + + - + + - + - - - - - - - - + -

- - - + - + + - - + + + + - - - + - + + + - - + - - - +

Represented in hexadecimal notation, the sequence is:

D651B4021678B91

The other sequence, in hex notation, is:

EC757781362D6F9

By starting with bit 52 of the second sequence and taking every 53rd bit modulo 60, one obtains the ones complement of the first sequence. Both sequences have SNR losses of only 0.15 dB when analyzed by the appropriate mismatched filter.

III. Implications for Aperiodic Sequences

An aperiodic sequence does not repeat. For it, the autocorrelation, $\boldsymbol{a}$, of a sequence, $\boldsymbol{C}$, of length ℓ is

$$a_i = \sum_{K=1}^{\ell-i} C_K C_{K+i}$$

The main lobe is a_0; the remaining a_i are sidelobes.

An aperiodic sequence is a Barker sequence if the absolute values of each of its sidelobes never exceeds 1. Aperiodic Barker sequences exist for all sequence lengths less than 6 as well as for lengths 7, 11, and 13.

The question of whether the aperiodic Barker sequence of length 13 is superior to aperiodic sequences of all other lengths has not yet been completely resolved. M. Golay has defined the "merit factor" for aperiodic sequences to be the square of the sequence length divided by twice the sum of the squares of the sidelobes (Ref. 6). His original upper bound for the merit factor was $2e^2 = 14.778$, which is approached only by the Barker-13 sequence's merit factor of 14.08. Golay states that 12.325 is a reasonable upper bound on the merit factor of all other aperiodic sequences. He has performed an exhaustive search of skew symmetric sequences of lengths up to 59 to support this. Whether or not the discovery of the present periodic sequence of length 60 stimulates further tests of Golay's bound, it should help to dispel the notion that the autocorrelation properties of the best periodic as well as aperiodic sequences degrade with increasing length.

References

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         IDENT    MVAL       Call assembler with ID
         ENTRY    MVAL       Define entry point
MVAL     EQ       $+1S17
         SB2      30         Put number of shifts into B2
         SB1      1          Used for decrementing
         SX5      30         For (number of 1's – 30)*2
         SX6      B0         Sum of squares of sidelobe
         SA2      X1         Put parameter value in X2
         BX3      X2         Put sequence to be shifted in X3
LOOP     LX3      1B         Left rotate the sequence by one bit
         BX4      X3–X2      XOR the shifted and original sequence
         CX4      X4         Count the ones in the result
         IX4      X4–X5      Subtract 30 from the number of 1's
         IX4      X4+X4      Double the difference
         IX4      X4*X4      Square the difference
         IX6      X6+X4      Add this to the total
         SB2      B2–B1      Decrement loop counter
         NE       B2,LOOP    Loop if not equal to 0
         IX6      X6+X6      Double the sum
         IX6      X6–X4      Avoid doubling middle term
         EQ       MVAL       This returns
         END
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Fig. 1. Sequence evaluator